SYLLABUS
For Ph.D. Course Work

Department of Mathematics
(Faculty of Science)

Ph.D. Program in Mathematics
### Course Structure

**A. Common Course (05 credits)**

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Title</th>
<th>Nature</th>
<th>Credits</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCC-01</td>
<td>Research Methodology and Computer</td>
<td>Compulsory</td>
<td>04</td>
</tr>
<tr>
<td></td>
<td>Application</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCC-02</td>
<td>Lab. Work based on SCC-01</td>
<td>Compulsory</td>
<td>01</td>
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**B. Discipline-Specific Courses (05 credits)**

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Title</th>
<th>Nature</th>
<th>Credits</th>
</tr>
</thead>
<tbody>
<tr>
<td>MA-601</td>
<td>Advancement in Mathematics</td>
<td>Compulsory</td>
<td>05</td>
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**C. Research Theme-Specific Courses (05 Credits)**

<table>
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<tr>
<th>Course Code</th>
<th>Title</th>
<th>Nature</th>
<th>Credits</th>
</tr>
</thead>
<tbody>
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<td>MA-602 to</td>
<td>Specialization paper (elective)</td>
<td>Elective</td>
<td>03</td>
</tr>
<tr>
<td>MA-620</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Review of literature &amp; Presentation</td>
<td></td>
<td>02</td>
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<table>
<thead>
<tr>
<th>Total Credit (A+B+C)</th>
<th>15</th>
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<tr>
<td>Duration of the entire course</td>
<td>06 month (i.e. One Semester)</td>
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1. Advancement in Mathematics (MA-601)

Unit-1:
Metric completion, Baire category theorem, Contraction mapping, Banach’s fixed point theorem, normed linear spaces, inequalities, Banach spaces, summability in Banach spaces.

Unit-2
Solution methods of finite difference equations: elliptic equations, parabolic equations, hyperbolic equation, example problems, stability, convergence and consistency of the solution methods, finite element methods.

Unit-3:
Method of weighted residuals: Galerkin’s method, collection method, method of least square, numerical solution of ordinary and partial differential equations using these methods.

Unit-4: MATLAB:
Basic Introduction: Simple arithmetic calculations, creating and working with arrays, numbers and matrices, creating and printing simple plots, function files, basic 2-D plots and 3-D plots. (05 Lectures)

Unit-5: LATEX:
Basic Introduction: Mathematical symbols and commands, Arrays, formulas, and equations, spacing, borders and colors, using date and time option in LaTeX, to create applications and letters, beamer (PPT in Latex), writing an article, pictures and graphics in LATEX.

Books:

2. Theory of Approximations (MA-602)

Unit-1.

Unit-2.
Carleson-Hunt theorem, Convergence of Fourier series, Divergence of Fourier, Weierstrass approximation theorem, Bernstein polynomials, Weierstrass second theorem, Monotone operators, Korovkin theorems, Modulus of continuity and its properties, Lipschitz class and its properties, Banach fixed point theorem, Jackson’s theorems,

Unit-3.
General linear families, characterization theorem, Haar conditions, Alteration theorem, Markoff systems, Theorem of de la Valle Poussin, Borel summability, Abel and matrix summability, Abel transformation, Product summability transform.

Unit-4.
Strong unicity theorem, Haar’s theorem, the convergence of Jackson theorems, Bernstein inequality, Bernstein theorems, Zygmund theorem, Summability theorems.

Unit-5.
Positive linear operators with their approximation properties, Local and Global approximation, Rate of convergence, weighted approximation, Direct and inverse results, Voronovskaja-type asymptotic formula, approximation in quantum calculus and (p, q)-calculus.

(Total Contact Time: 50 Hours)

BOOKS:
3. Applied Summability Methods (MA-603)

Unit 1.
Sequence, infinite series, summability factors, Abel summability and Cesàro summability, Euler-Knopp matrix, Nörlund mean, Borel matrix, Riesz mean, The Dirichlet and Fejér kernels, Matrix transformations.

Unit 2.
Toeplitz matrix, Regularity of summability methods, almost strongly regular matrices, Banach limits and almost convergence, Absolute almost convergence, Some inclusion theorems for sequence spaces. Absolute indexed Nörlund and Riesz summability.

Unit 3.
Lambert summability and the prime number theorem, Hausdorff matrices, Nonlinear summability, Almost summability of Taylor series, Matrix summability of Fourier and Walsh Fourier series.

Unit 4.
Summability tests for singular points, Lototski summability and analytic continuation, summability methods for random variables, A-summability of a sequence of random variables, application of almost convergence in approximation theorems for functions of two variables.

Unit 5.

(Total Contact Time: 50 Hours)

BOOKS:
4. Nonlinear Analysis and Applications (MA-604)

Unit 1.
Nonlinear operators, monotone, strictly monotone and strongly monotone operators, their properties and applications. Calculus of Banach space, Frechet and Gateaux differentiability.

Unit 2.

Unit 3.
Fixed point theory, Banach contraction mapping theorem, Non-expansive mappings, contractive type mappings, generalizations of Banach contraction mapping theorem, fixed point theorem of other types, Applications, Optimization in Banach spaces.

Unit 4.

Unit 5.

(Total Contact: 50 Hours)

Books:
5. Optimization Theory and Applications (MA-605)

Unit 1: Linear Programming Problem
Linear Programming Problem (LPP), Requirements of LPP, Mathematical formulation of LPP, Examples from industrial cases, Two-Variable LP Model, Graphical LP Solution, Solution of a Maximization Model, Solution of a Minimization Model, Selected LP Applications, Advantages, Limitations, Motivation of the simplex method, Simplex method, Penalty cost method or Big M-method, Two phase method, Importance of duality concepts, Formulation of dual problem, Economic interpretation of duality, Dual simplex method.

Unit 2: Integer Programming
Motivation of Integer Programs, Formulation of various industrial problems as integer and mixed integer programming problems, Branch and bound algorithm, Cutting plane method.

Unit 3: Dynamic Programming
Multistage decision processes, Concept of Bellman’s principle of optimality and recursive relationship of dynamic programming for different optimization problems.

Unit 4: Network Optimization Models

Unit 5: Nonlinear Programming
Types of nonlinear programming problems, Differentiable convex function, Minimization and maximization, Convex set, Convex function, Differentiable convex functions, Sub differential of a convex function, Saddle point Conditions, Single variable optimization; Multi variable optimization with no constraints (semidefinite case, saddle point), with equality constraints (solution by direct substitution, method of constrained variation, method of Lagrange multipliers), with inequality constraints (Kuhn-Tucker conditions, constraint qualification); Convex programming problem, NLP: One dimensional minimization methods.

(Total Contact time: 50 Hours)

Books:
6. Actuarial Mathematics (MA-606)

Unit-1:
Discrete random variables, some discrete probability distributions, discrete uniform, binomial, negative binomial, geometric and poisson distribution, some continuous probability distributions, continuous uniform, normal, exponential, and gamma distribution. (Lecture 10)

Unit-2:
Probability for the Age-at-Death, the survival function, time- until-death for a person aged \(x\), Curtate-future-lifetime, force of mortality, life tables, relation of life table functions to the survival function, life table examples, the deterministic survivorship group, other life table functions, assumptions for fractional ages, Some analytical laws of mortality, some analytical laws of mortality, select and ultimate tables.(Lecture 10)

Unit-3:
Introduction to Life Insurance, insurances payable at the moment of death, level benefit insurance, endowment insurance, deferred insurance, varying benefit insurance, insurances payable at the end of year of death, relationships between Insurances payable at the moment of death and the end of year of death, recursion equation, commutation functions.(Lecture 10)

Unit-4:
Single payment contingent on survival, continuous life annuities, discrete life annuities, life annuities with mthly payments, commutation function formulas for annuities with level payments, varying annuities, recursion equations, complete annuities-immediate and apportionable annuities-due. (Lecture 10)

Unit-5:
Net premiums or benefit premiums, the random future loss under an assurance or annuity contract, state the principle of equivalence, notations and formulae of net premium for common life insurance contracts, fully discrete premiums, true monthly payment premium, commutation functions, increasing and decreasing benefit premiums, profits contract, types of bonus, calculating net premiums for with-profit contracts.(Lecture 10)

Books:

7. Design and Analysis of Algorithms (MA-607)

Unit-1:
Algorithm introduction, algorithm specification, pseudo code conventions, recursive algorithms, performance analysis, space complexity, time complexity, asymptotic notation, practical complexities. (Lecture 10)

Unit-2:
Data structures and Queues, linear data structures, concepts of non-primitive data structures, storage structure for arrays, stacks, operations on stacks, queues, priority queues. (Lecture 10)

Unit-3:
Linked lists and trees, linked linear lists, operations on linked linear lists, circularly linked lists, doubly linked linear lists, non-linear data structures, trees, binary trees, operations on binary trees, storage representation and manipulations of binary trees. (Lecture 10)

Unit-4:
Search and Sort, Divide and conquer, general method, binary search, finding the maximum and minimum in a set of items, merge sort, quick sort, selection sort, basic traversal and search techniques for graphs, breadth first search, depth first search. (Lecture 10)

Unit-5:
Backtracking, the 8-Queens problem, algebraic problems, the general method, evaluation and interpolation, Horner’s rule, Lagrange interpolation, Newtonian interpolation. (Lecture 10)

Books:
8. Soft Computing (MA-608)

Unit-1:
Introduction to soft computing, evolution of computing, Soft Computing constituents, from conventional Artificial Intelligence to computational Intelligence, Machine Learning basics. (Lecture 8)

Unit-2:
Introduction to Genetic Algorithms (GAs), building block hypothesis, working principle, basic operators and terminologies like individual, gene, encoding, fitness function and reproduction, Genetic modelling, significance of Genetic operators, Inheritance operator, cross over, inversion & deletion, mutation operator, bitwise operator, GA optimization problems, JSPP (Job Shop Scheduling Problem), TSP (Travelling Salesman Problem), Differences & similarities between GA & other traditional methods, Applications of GA. (Lecture 14)

Unit-3:

Unit-4:
Fuzzy Sets, operations on fuzzy sets, fuzzy relations, membership functions, fuzzy rules and fuzzy reasoning, fuzzy inference systems, fuzzy expert systems, fuzzy decision making. (Lecture 8)

Unit-5:

Books:
9. Energy Aware Computing (MA-609)

Unit-1:
Energy efficient network on chip architecture for multi core system, energy efficient MIPS CPU core with fine grained run time power gating, low power design of emerging memory technologies. (Lecture 10)

Unit-2:
Disk energy management, power efficient strategies for storage system, dynamic thermal management for high performance storage systems, energy saving technique for disk storage systems. (Lecture 10)

Unit-3:
Scheduling of parallel tasks, task level dynamic voltage scaling, speed scaling, processor optimization, memeticalgorithms, and online job scheduling Algorithms. (Lecture 10)

Unit-3:
Multi-processor system, real time tasks, energy minimization, energy aware scheduling, dynamic reconfiguration, adaptive power management, energy harvesting embedded system. (Lecture 10)

Unit-3:
Energy aware applications: on chip network, video codec design, surveillance camera, low power mobile storage. (Lecture 10)

Books:
10. Intelligent Computing (MA-610)

Unit-1:

Unit-2:
Neural Networks: mathematical model of neural networks, artificial neural network learning methods and learning strategies, activation functions, multilayer-perceptron network, Selforganizing Map (Kohonen network), Hopfield Network, Radial Basis Function (RBF) network. (Lecture 10)

Unit-3:
Fuzzy Logic: Crisp set and Fuzzy set, basic concepts of fuzzy sets, membership functions, basic operations on fuzzy sets, properties of fuzzy sets, fuzzy relation, hybrid system and its applications. (Lecture 10)

Unit-4:
Computational Logic: modal logic and temporal logic, some applications of modal logic and temporal Logic, multi agent systems, agent and their characteristics, multi agent paradigm, coordination and communication and cooperation. (Lecture 10)

Unit-5:
Application to intelligent tutoring systems, E-commerce and Elearning. (Lecture 10)

Books:

11. Parallel Computing (MA-611)

Unit 1:
Review of multiprocessor and distributed systems, Conditions of parallelism, program partitioning and program flow mechanisms. Parallel Models: Shared memory model, message memory model, data parallel model, object-oriented model, functional and logic models. (Lecture 10)

Unit 2:
Parallel Algorithms: Cost, Efficiency, PRAM algorithms, Mesh algorithms, hypercube algorithms, combinational circuit algorithms. (Lecture 10)

Unit 3:
Parallel languages and compilers: Language features for parallelism, parallel language constructs, optimizing compilers for parallelism, dependency analysis, code optimization and scheduling, loop parallelization and pipelining. (Lecture 10)

Unit 4:
Parallel program development: Parallel programming environments, synchronization and multiprocessing modes, shared variable program structures, message passing, program development, mapping programs onto, multicomputers. (Lecture 10)

Unit 5:
Multiprocessor UNIX (design goals), Master slave and multithreaded Unix, multicomputer Unix extension, Mach/OS kernel architecture, OSF/1 architecture and programming environment. (Lecture 10)

Books:
12. Mathematical modeling for Health Care (MA-612)

Unit 1:
Introduction to health care information, health care data quality, health care information regulations, laws and standards. (Lecture 8)

Unit 2:
History and evolution of health care information systems, current and emerging use of clinical information systems, system acquisition, system implementation and support. (Lecture 10)

Unit 3:
Information architecture and technologies that support health care information systems, health care information system standards, security of health care information systems. (Lecture 10)

Unit 4:
Management of IT challenges: Organizing information technology services — IT alignment and strategic planning, IT governance and management. (Lecture 10)

Unit 5:
IT Initiatives: Management’s role in major IT initiatives, Assessing and achieving value in health care information systems. Case study. (Lecture 12)

Books:
13. Problem Solving and Programming (MA-613)

Unit 1:
Introduction: the problem solving aspect, top down design, implementation of algorithm, program verification, the efficiency of algorithm, the analysis of algorithm. (Lecture 8)

Unit 2:
Programs and programming, building blocks for simple programs, programming life cycle phases, pseudo code representation, flow charts, algorithm, programming languages, compiler, interpreter, loader and linker, program execution, classification of programming language, structured programming concept. (Lecture 10)

Unit 3:
Identifier, keywords, variables, constants – I/O statements, operators, initialization, expressions, expression evaluation, Lvalues and Rvalues, type conversion in C, formatted input and output functions, specifying test condition for selection and iteration, conditional execution and selection, iteration and repetitive execution, go to Statement, Nested Loops- Continue and break statements. (Lecture 10)

Unit 4:
Array – One dimensional character arrays, multidimensional arrays- arrays of strings, two dimensional character array, functions, parameter passing mechanism scope, storage classes, recursion, comparing iteration and recursion, pointers, pointer operators, uses of pointers, arrays and pointers, pointers and strings, pointer indirection, pointers to functions, dynamic memory allocation. (Lecture 12)

Unit 5:
Structures, initialization, nested structures, structures and arrays, structures and pointers, union, typedef and enumeration types, bit fields, File Management in C, Files and Streams, file handling functions, sequential access file, random access file, command line arguments. (Lecture 10)

Books:
4. Deitel and Deitel, C How to Program, Pearson Education. 2010.
14. Artificial Intelligence (MA-614)

Unit-1:

General Issues and Overview of AI: The AI problems, what is an AI technique, characteristics of AI applications. Introduction to LISP programming: syntax and numeric functions, basic list manipulation functions, predicates and conditionals, input output and local variables, iteration and recursion, property lists and arrays. (Lecture 10)

Unit-2:


Unit-3:

Knowledge representations first order predicate calculus, skolemization, resolution principle & unification, interface mechanisms, horn's clauses, semantic networks, frame systems and value inheritance, scripts, conceptual dependency. (Lecture 10)

Unit-4:

Natural language processing parsing techniques, context free grammer, recursive transitions nets (RNT), augmented transition nets (ATN), case and logic grammers, symantc analysis. game playing minimax search procedure, alpha-beta cutoffs, additional refinments, planning overview an example domain the block word, component of planning systems, goal stack planning, nonlinear planning. (Lecture 10)

Unit-5:

Probabilistic reasoning and uncertainty probability theory, bayes theorem and bayesian networks, certainty factor, expert systems introduction to expert system and application of expert systems, various expert system shells, vidwan frame work, knowledge acquisition, case studies, MYCIN, learning rote learning, learning by induction, explanation based learning. (Lecture 10)

Books:

15. Mathematical Control Theory (MA-615)

Prerequisite: Linear Algebra, Functional Analysis

Unit 1:

Linear differential equations, finite-dimensional linear control systems, transition matrix, controllability matrix, Kalman rank condition, observability.

Unit 2:

Stability, stable polynomial, Routh theorem, Liapunov equation, stabilizability and controllability.

Unit 3:

Infinite-dimensional linear control systems, introduction to semigroups of linear operators, controllability operator, stability, Liapunov equation, stabilizability and controllability.

Unit 4:

Nonlinear control systems, linearization, controllability, stability, Liapunov function, La Salle’s theorem, topological stability

Unit 5:

Optimal control problems, introduction to calculus of variations, maximum principle.

Text Books:


Reference Books:


Prerequisites: Theory of Ordinary and Partial Differential Equations, Functional Analysis

Unit 1:

Semigroup of bounded linear operators, infinitesimal generator, Hille-Yoshida theorem, Lumer-Phillips theorem, pseudo resolvents, adjoint semigroup, semigroup of compact operators.

Unit 2:

Differentiability, analytic semigroups, contraction semigroups, fractional powers of closed operators, perturbation of: bounded linear operators, infinitesimal generator of analytic and contraction semigroups.

Unit 3:

Abstract Cauchy problems, homogeneous and inhomogeneous initial value problems, classical solution, mild solution, regularity and asymptotic behaviour of solutions, invariant and admissible subspaces.

Unit 4:

Evolutions equations, stable family of generators, evolution systems in hyperbolic and parabolic cases, regularity and asymptotic behaviour of solutions, Lipschitz perturbation of evolution equations, semilinear equations with compact and analytic semigroups, quasilinear evolution equations.

Unit 5:

Applications to partial differential equations: heat equation, wave equation, Schrodinger equation, Korteweg-de Vries equation.

Books:

17. Applicable Analysis (MA-617)

Prerequisites: Functional Analysis

Unit 1:

Reflexivity, weak convergence, dual spaces, self-adjoint operators, compact operators, monotone operators, positive operators, accretive operators, unbounded operators.

Unit 2:

Spectrum and resolvent sets, spectral theory for bounded and unbounded operators, perturbation theory.

Unit 3:

Fixed point theorems due to Banach, Brouwer, Schauder, Schaefer, Krasnoselskii, Leray-Schauder, Kakutani.

Unit 4:

Test functions and distributions, support and singular support, convolution of distributions, Fourier transform, Schwartz space, Fourier inversion formula, tempered distribution.

Unit 5:

Sobolev spaces definition and basic properties, approximation by smooth functions, extension theorem, imbedding theorem, compactness theorem, dual spaces, fractional order spaces, trace spaces.

Books:

4. D.R. Smart, Fixed Point Theorems, Cambridge University Press, 1974. (Unit 3)
18. **Dynamical Systems (MA-618)**

Prerequisite: Systems of Linear Equations

**Unit 1:**

Dynamical systems and vector fields, fundamental theorem of ordinary differential equations, existence and uniqueness of solution, continuity of solutions in initial conditions, extending solutions, global solutions.

**Unit 2:**

Flow of a differential equation, nonlinear sinks, stability of equilibrium, stability theory, Liapunov functions, gradient systems.

**Unit 3:**

Limit sets, attractors, periodic orbits, limit cycles, separatrix cycles, monotone sequences in planar dynamics.

**Unit 4:**

Poincare map, Poincar-Bendixson theorem, Lienard systems, Poincare sphere and behaviour at infinity.

**Unit 5:**

Differential equations in electrical circuits, Van der Pol’s equation, Hopf bifurcation, bifurcation analysis for: one species, prey-predator species, competing species. Introduction to discrete dynamical systems.

**Books:**

19. Fluid Mechanics (MA-619)

Unit 1:
Kinematics of fluid in motion: real fluids and ideal fluids, stream lines, streak lines and path lines, steady and unsteady flows, velocity potential, vorticity vector, Lagrangian and Eulerian description, continuity of mass flow (different coordinates also), circulation, rotational and irrotational flows, boundary surface, Reynolds transport theorem, Bernoulli’s theorem, compressible and incompressible flows, Kelvin’s theorem.

Unit 2:
Stress and strain and relation between stress and strain, Stokes hypothesis, formulation of the Navier-Stokes equations, special forms of Navier-Stokes equations: Stokes equations and Euler equations, classification of partial differential equations and physical behavior,

Unit 3:
Fully developed flows with examples, some solvable problem in viscous flow: (i) steady flow between parallel plates, (ii) steady flow in a pipe, (iii) steady flows between concentric cylinders (iii) Couette flow, derivation of energy equation, dissipation of energy, diffusion of vorticity, Boussinesq approximation.

Unit 4:
Dimensional analysis, physical signification of some non-dimensional parameters: Prandtl, number, Mac number, Reynolds number, Froud number, Reyleigh number, Grashof number, Nusselt number etc.

Concept of boundary layers, boundary layer thickness, Prandtts boundary layer, similarity solution, boundary layer on flat plate: Blassius solution, von-Karman momentum integral equation, Effect of pressure gradient, wall shear stress, separation of the boundary layer.

Unit 5:

Books:
20. Computational Fluid Dynamics (MA-620)

Unit 1:
Brief introduction of ODE (IVP and BVP) and PDE, initial and boundary conditions, classification of PDE, kinematics of fluid in motion, discussion about experimental, theoretical and numerical approaches in fluid mechanics, mathematical description of the physical phenomena, governing equations mass, momentum, energy, classification of partial differential equations, physical classification, mathematical classification, well-posed problems, some solvable problem in viscous flow: (i) steady flow between parallel plates, (ii) steady flow in a pipe.

Unit 2:
Physical description of FDM, derivation of finite differences formulas with high accuracy, multidimensional formulas, accuracy of finite difference solutions, solution methods of finite difference equations: elliptic equations, parabolic equations, hyperbolic equation, example problems, stability, convergence and consistency of the solution methods.

Unit 3:
ADI-method, application of finite difference methods to the equations of fluid mechanics, vorticity formulation: solution of viscous incompressible flows by the stream function.

Unit 4:
Galerkin’s method, collocation method, method of least square, numerical solution of ordinary and partial differential equations using these methods, introduction of finite element method and basic difference between finite element and finite difference method, linear and quadratic and higher order elements in one dimensional, assembly of elements, solution of ODE (related to fluid mechanics) by FEM.

Unit 5:
Introduction to finite volume methods: basic formulations, finite-volume method for diffusion problems.

Books: